## Solving IV Flow Rate Problems Using Dimensional Analysis

Terminology:

- Dimensional Analysis (DA): A powerful method of solving problems in pharmacy, chemistry, physics, and engineering in which a given is multiplied by one or more ratios to obtain the answer.
- IV: Abbreviation for intravenous, meaning administered into a vein.
- drop factor: The number of drops (gtts) per mL. Macrodrip tubing comes 10, 15, $20 \mathrm{gtts} / \mathrm{mL}$ while microdrip tubing is $60 \mathrm{gtt} / \mathrm{mL}$.
- flow rate/infusion rate/drip rate: The volume of solution or weight of drug delivered over time. The units are usually gtts/min, $\mathrm{mL} /$ hour or $\mathrm{mg} /$ hour.

These problems all have the same three parts:

- The Units of the Answer: Think of it as the destination.
- A Given: This is what is given to start the problem and what is changed into the answer.
- One or More Ratios: These are the tools used to change the units of the given into the units of the answer.


## Example of a simple dimensional analysis problem: Convert 4.5 g into mg .

- The units of the answer are mg . This is the destination.
- The given is 4.5 g . This is the starting point.
- The ratio is $1000 \mathrm{mg} / \mathrm{g}$. This is the tool to change g to mg .
- Start by listing the starting point and destination. This will help when placing the ratio(s).

$$
4.5 \mathrm{~g}=\mathrm{mg}
$$

- Place the ratio with the units of the answer on top and the units to be canceled on the bottom. Multiply the given by the ratio. The grams cancel out, leaving mg in the answer.

$$
4.5 \mathrm{~g}\left(\frac{1000 \mathrm{mg}}{\mathrm{~g}}\right)=4500 \mathrm{mg}
$$

Example of a dimensional analysis problem involving a numerator and denominator: If an object is traveling 50 ft per minute, how many inches is it traveling per second?

$$
\frac{50 \mathrm{ft}}{\min }=\frac{\mathrm{in}}{\sec }
$$

On this one, you have to change feet to inches and minutes to seconds. 1 foot = 12 in and $1 \mathbf{~ m i n ~}=\mathbf{6 0} \mathbf{~ s e c}$.

$$
\frac{50 \mathrm{ft}}{\min }\left(\frac{12 \mathrm{in}}{\mathrm{ft}}\right)\left(\frac{1 \mathrm{~min}}{60 \mathrm{sec}}\right)=10 \frac{\mathrm{in}}{\sec }
$$

## The general setup of an IV flow rate problem is:

(Given)(Ratios) = Answer

The problem will supply you with the given and the units of the answer. The ratios will either be supplied in the problem, or you may have to use your own ( $60 \mathrm{~min} / \mathrm{h}, 1000 \mathrm{mcg} / \mathrm{mg}$, etc.).

There are three types of IV Flow rate problems:

- The rate to rate problem is the most common. For example: An IV is running at a rate of 1 Liter per hour with a drop factor of 20 ( $20 \mathrm{drops} / \mathrm{mL}$ ). What is the rate in drops/min? If the units of the answer are a rate, the given must be a rate.
(Rate)(Ratios) = Rate
- The time to quantity problem will give you a time duration and ask for the quantity of something ( $\mathrm{mL}, \mathrm{mg}, \mathrm{mg} / \mathrm{kg}, \mathrm{mEq}$ ) delivered over that duration. For example: An IV with a flow rate of $500 \mathrm{~mL} / \mathrm{h}$ has been running for 2 hours. What volume of fluid has been administered?
If the units of the answer are a quantity, the given must be a time duration.
(Time)(Ratios) = Quantity
- The quantity to time is just the opposite of the time to quantity problem. You will be given a quantity of something and asked for the time duration to administer it. For example: How long will it take to administer 1 L of fluid at the rate of $250 \mathrm{~mL} / \mathrm{h}$ ?
If the units of the answer are a time duration, the given must be a quantity.
(Quantity)(Ratios) = Time


## Examples:

 Calculate the flow rate in gtts/min.

Step 1) Look at the units of the answer. Gtts/min is a rate, so the given must be a rate. The only other rate in the problem is $1000 \mathrm{~mL} / 4 \mathrm{~h}$. Write these down with an equal sign.

$$
\frac{1000 \mathrm{~mL}}{4 \mathrm{~h}} \quad=\frac{\mathrm{gtts}}{\mathrm{~min}}
$$

Now you can see that you have to change mL to gtts and h to min . The ratio of $\frac{10 \mathrm{gtts}}{\mathrm{mL}}$ will change mL to gtts and the ratio $\frac{1 \mathrm{~h}}{60 \mathrm{~min}}$ will change hours to minutes.

Step 2) Arrange the ratios so the unwanted units cancel leaving the units of the answer.

$$
\frac{1000 \mathrm{~mL}}{4 \mathrm{~h}}\left(\frac{10 \mathrm{gtts}}{\mathrm{~mL}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=\frac{\mathrm{gtts}}{\mathrm{~min}}
$$

Step 3) Cross out the units which cancel and make sure that only the units of the answer remain.

$$
\frac{1000 \mathrm{~mL}}{4 \mathrm{~h}}\left(\frac{10 \mathrm{gtts}}{\mathrm{~mL}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=\frac{\mathrm{gtts}}{\mathrm{~min}}
$$

Step 4) Take out your calculator and do the calculations. Multiply everything on top and divide by everything on the bottom, giving the answer of $41.7 \mathrm{gtts} / \mathrm{min}$ which is rounded to 42 gtts/min.
2) A patient has an order for regular insulin at the rate of 18 units/hour. The solution is 100 mL with 100 units of regular insulin. An infusion set with a drop factor of 20 is being used. What will be the flow rate in gtts/min?

Step 1) Looking at the units of the answer you see gtts/min, so you know the given must be a rate. The only other rate in the problem is 18 units/hour, so you know this is the given.

$$
\frac{18 \text { units }}{h} \quad=\frac{\text { gtts }}{\min }
$$

You will have to change units to gtts and hours to minutes. It will take two ratios to change units to gtts, $\frac{100 \mathrm{~mL}}{100 \mathrm{units}}$ and $\frac{20 \mathrm{gtts}}{\mathrm{mL}}$. The ratio of $\frac{1 \mathrm{~h}}{60 \mathrm{~min}}$ will change hours to minutes.

Step 2) Arrange the ratios so the unwanted units cancel leaving the units of the answer. Double check everything and do the calculations.

$$
\frac{18 \text { units }}{\mathrm{h}}\left(\frac{100 \mathrm{~mL}}{100 \text { units }}\right)\left(\frac{20 \mathrm{gtts}}{\mathrm{~mL}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=6 \frac{\mathrm{gtts}}{\mathrm{~min}}
$$

3) A patient has an order for a drug to be infused at the rate of $25 \mathrm{mg} / \mathrm{kg} / \mathrm{h}$. A 1 L bag contains 10 g of the drug and the patient weighs 80 kg . An infusion set with a drop factor of 15 is being used. What is the flow rate in gtts/min?

This problem looks a little different because it contains the rate $\mathbf{2 5 ~ m g} / \mathrm{kg} / \mathrm{h}$. This means
$(25 \mathrm{mg} / \mathrm{kg}) / \mathrm{h}$. You can either enter the rate as $\frac{25 \mathrm{mg} / \mathrm{kg}}{\mathrm{h}}$ or my prefered way $\frac{25 \mathrm{mg}}{\mathrm{kg} \mathrm{h}}$, which is mathematically equivalent.

Step 1) Look at the units of the answer. Gtts/min is a rate, so the given must be a rate. The only other rate in the problem is $25 \mathrm{mg} / \mathrm{kg} / \mathrm{h}$. Write these down with an equal sign.

$$
\frac{25 \mathrm{mg}}{\mathrm{~kg} \mathrm{~h}} \quad=\frac{\mathrm{gtts}}{\min }
$$

You will have to change mg to gtt and h to minutes. kg is not changed to anything, but rather eliminated from the equation. The patient's weight is part of the given and will be inserted above the line to eliminate kg .

Step 2) Arrange the ratios and the patient's weight so the unwanted units cancel leaving the units of the answer. Double check everything and do the calculations.

$$
\frac{25 \mathrm{mg}}{\mathrm{~kg} \mathrm{~h}}\left(\frac{80 \mathrm{~kg}}{}\right)\left(\frac{1 \mathrm{~L}}{10 \mathrm{~g}}\right)\left(\frac{1 \mathrm{~g}}{1000 \mathrm{mg}}\right)\left(\frac{1000 \mathrm{~mL}}{\mathrm{~L}}\right)\left(\frac{15 \mathrm{gtts}}{\mathrm{~mL}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=50 \frac{\mathrm{gtts}}{\mathrm{~min}}
$$

4) Calculate the length of time required to infuse a 1000 mL bag at the rate of $50 \mathrm{~mL} / \mathrm{h}$.

Step 1) Look at the units of the answer. Although it doesn't say "Calculate the number of hours", you can figure that out yourself. Since the units of the answer is a time duration, you know that the given must be a quantity. The only quantity in the problem is 1000 mL .

$$
1000 \mathrm{~mL} \quad=\text { hours }
$$

Step 2) Whenever you have a time to quantity or quantity to time problem, one of the ratios must be a rate. The only rate in the problem is $50 \mathrm{~mL} / \mathrm{h}$, so you know that must be part of the equation.

$$
1000 \mathrm{~mL}\left(\frac{1 \mathrm{~h}}{50 \mathrm{~mL}}\right)=20 \mathrm{~h}
$$

5) An IV has been running for 2 hours at the rate of $40 \mathrm{~mL} / \mathrm{h}$. How many mL have been administered?

This is an example of a simple time to quantity problem. The units of the answer are mL , so the given must be a time duration.

$$
2 \mathrm{~h}=\mathrm{mL}
$$

The rate of $40 \mathrm{~mL} / \mathrm{h}$ will change h to mL .

$$
2 \mathrm{~h}\left(\frac{40 \mathrm{~mL}}{\mathrm{~h}}\right)=80 \mathrm{~mL}
$$

## Summary

1) Look at the units of the answer.

- If it is a rate, the given will be a rate.
- If it is a duration of time, the given will be a quantity.
- If it is a quantity, the given will be a duration of time.

2) Compare the given to the units of the answer.
3) Insert the ratios so the unwanted units cancel leaving the units of the answer.
4) Double check everything and do the calculations.

## The Typical Thought Process of a Student Working an IV Flow Rate Problem using Dimensional Analysis

A 59 y.o. male weighing 80 kg has been admitted to the ED and placed on a dopamine infusion at the rate of $45 \mathrm{~mL} / \mathrm{h}$. The IV bag contains 400 mg of dopamine in $\mathbf{2 5 0} \mathbf{~ m L}$ of D5W (5 g of dextrose/ 100 mL solution). The IV has been running for $\mathbf{2 0}$ minutes. How many mcg/kg of dopamine is the patient receiving each minute?

Student: Oh brother. Someone is trying to be cute by loading this problem up with a lot of crap. What are they asking? $\mathrm{mcg} / \mathrm{kg} / \mathrm{min}$, ok, that is a rate so the problem has to start with a rate. The only other rate is $45 \mathrm{~mL} / \mathrm{h}$. Glad I read somewhere that $\mathrm{mcg} / \mathrm{kg} / \mathrm{min}=\mathrm{mcg} /(\mathrm{kg} \mathrm{min})$. I hate those two line things.

$$
\frac{45 \mathrm{~mL}}{\mathrm{~h}} \quad=\frac{\mathrm{mcg}}{\mathrm{~kg} \mathrm{~min}}
$$

Student: That was a fun party last night. Nice of Becky to invite me to her dog's tenth birthday party. Concentrate. I have to change mL to mcg , h to min and add kg to the answer. I will start with something easy.

$$
\frac{45 \mathrm{~mL}}{\mathrm{~h}}\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right) \quad=\frac{\mathrm{mcg}}{\mathrm{~kg} \mathrm{~min}}
$$

Student: I probably shouldn't have had that last beer. What is the maximum dose of ibuprofen? Concentrate...There is $\mathbf{4 0 0} \mathbf{~ m g}$ of dopamine in $\mathbf{2 5 0 ~ m L}$. mL will have to go on the bottom to cancel out the $\mathbf{m L}$ in 45 mL .

$$
\frac{45 \mathrm{~mL}}{\mathrm{~h}}\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)\left(\frac{400 \mathrm{mg}}{250 \mathrm{mb}}\right) \quad=\frac{\mathrm{mcg}}{\mathrm{~kg} \mathrm{~min}}
$$

Student: Did I really fill the dog's bowl up with beer? mg to mcg is easy.

$$
\frac{45 \mathrm{~mL}}{\mathrm{~h}}\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)\left(\frac{400 \mathrm{mg}}{250 \mathrm{~mL}}\right)\left(\frac{1000 \mathrm{mcg}}{\mathrm{mg}}\right) \quad=\frac{\mathrm{mcg}}{\mathrm{~kg} \mathrm{~min}}
$$

Student: He probably didn't drink it. What if he did? What if he got sick? Damn..I do stupid stuff when I drink. I just have to add the kg on the bottom. He weighs 80 kg. .that is easy.

$$
\frac{45 \mathrm{~mL}}{\mathrm{~h}}\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)\left(\frac{400 \mathrm{mg}}{250 \mathrm{~mL}}\right)\left(\frac{1000 \mathrm{mcg}}{\mathrm{mg}}\right)\left(\frac{1}{80 \mathrm{~kg}}\right) \quad=\frac{\mathrm{mcg}}{\mathrm{~kg} \mathrm{~min}}
$$

Student: Concentrate...mL, h, mg all cancel out. Smart of me to highlight the things I want. Looks good. Get out the calculator. $15 \mathrm{mcg} / \mathrm{kg} / \mathrm{min}$. I better use the two line thing so I don't confuse the instructor. I better call Becky and make sure her dog is ok.

## IV Flow Rate Exercise

Calculate the flow rate in $\mathrm{mL} / \mathrm{h}$.

1) 1000 mL infused over 5 h .
2) 250 mL infused over 2 h .

Calculate the flow rate in gtts/min. Round to the nearest drop.
3) 1000 mL infused over 4 hours using an infusion set with a drop factor of 10 ( $10 \mathrm{gtts} / \mathrm{mL}$ ).
4) 250 mL infused over 2 hours using an infusion set with a drop factor of 15 .
5) 2 L infused over 24 hours using an infusion set with a drop factor of 20.
6) 100 mL infused over 1 hour using an infusion set with a drop factor of 10 .
7) 1000 mL infused over 5 hours using an infusion set with a drop factor of 20.

Calculate the length of time required to infuse the following volumes.
8) A 1000 mL bag infused at the rate of $45 \mathrm{~mL} / \mathrm{h}$.
9) A 1000 mL bag infused at the rate of $45 \mathrm{~mL} / \mathrm{h}$ using an infusion set with a drop factor of 20 .
10) A 1000 mL bag infused at the rate of $45 \mathrm{~mL} / \mathrm{h}$ using an infusion set with a drop factor of 10 .
11) A 1 L bag infused at the rate of $50 \mathrm{gtts} / \mathrm{min}$ using an infusion set with a drop factor of 15 .
12) A 500 mL bag infused at the rate of $25 \mathrm{gtts} / \mathrm{min}$ using an infusion set with a drop factor of 20.

## Answer the following:

13) A patient has an order for regular insulin at the rate of 18 units/hour. The solution is 100 mL with 100 units of regular insulin. An infusion set with a drop factor of 20 is being used. What will be the flow rate in gtts/min?
14) A patient has an order for a drug to be infused at the rate of $5 \mathrm{mcg} / \mathrm{kg} / \mathrm{min}$. A 500 mL bag contains 250 mg of the drug and the patient weighs 185 pounds. An infusion set with a drop factor of 20 is being used. What is the flow rate in $\mathrm{gtts} / \mathrm{min}$ ?
15) A patient has an order for a drug to be infused at the rate of $25 \mathrm{mg} / \mathrm{kg} / \mathrm{h}$. A 1 L bag contains 10 g of the drug and the patient weighs 80 kg . An infusion set with a drop factor of 15 is being used. What is the flow rate in $\mathrm{gtts} / \mathrm{min}$ ?

## IV Flow Rate Exercise Answers

Calculate the flow rate in $\mathrm{mL} / \mathrm{h}$.

1) 1000 mL infused over 5 h .

$$
\frac{1000 \mathrm{~mL}}{5 \mathrm{~h}}=\frac{200 \mathrm{~mL}}{\mathrm{~h}}
$$

2) 250 mL infused over 2 h .

$$
\frac{250 \mathrm{~mL}}{2 \mathrm{~h}}=\frac{125 \mathrm{~mL}}{\mathrm{~h}}
$$

Calculate the flow rate in gtts/min. Round to the nearest drop.
3) 1000 mL infused over 4 hours using an infusion set with a drop factor of 10 ( $10 \mathrm{gtts} / \mathrm{mL}$ ).

$$
\frac{1000 \mathrm{~mL}}{4 \mathrm{~h}}\left(\frac{10 \mathrm{gtts}}{\mathrm{mb}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=\frac{42 \mathrm{gtts}}{\mathrm{~min}}
$$

4) 250 mL infused over 2 hours using an infusion set with a drop factor of 15 .

$$
\frac{250 \mathrm{~mL}}{2 \mathrm{~h}}\left(\frac{15 \mathrm{gtts}}{\mathrm{mt}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=\frac{31 \mathrm{gtts}}{\mathrm{~min}}
$$

5) 2 L infused over 24 hours using an infusion set with a drop factor of 20.

$$
\frac{2 \mathrm{~L}}{24 \mathrm{~h}}\left(\frac{1000 \mathrm{~mL}}{\mathrm{~L}}\right)\left(\frac{20 \mathrm{gtts}}{\mathrm{~mL}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=\frac{28 \mathrm{gtts}}{\mathrm{~min}}
$$

6) 100 mL infused over 1 hour using an infusion set with a drop factor of 10 .

$$
\frac{100 \mathrm{~mL}}{1 \mathrm{~h}}\left(\frac{10 \mathrm{gtts}}{\mathrm{~mL}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=\frac{17 \mathrm{gtts}}{\min }
$$

7) 1000 mL infused over 5 hours using an infusion set with a drop factor of 20 .

$$
\frac{1000 \mathrm{~mL}}{5 \mathrm{~h}}\left(\frac{20 \mathrm{gtts}}{\mathrm{mb}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=\frac{67 \mathrm{gtts}}{\mathrm{~min}}
$$

Calculate the length of time required to infuse the following volumes.
8) A 1000 mL bag infused at the rate of $45 \mathrm{~mL} / \mathrm{h}$.

$$
1000 \mathrm{~mL}\left(\frac{1 \mathrm{~h}}{45 \mathrm{~mL}}\right)=22.2 \mathrm{~h}
$$

9) A 1000 mL bag infused at the rate of $45 \mathrm{~mL} / \mathrm{h}$ using an infusion set with a drop factor of 20.

$$
1000 \mathrm{~mL}\left(\frac{1 \mathrm{~h}}{45 \mathrm{~mL}}\right)=22.2 \mathrm{~h}
$$

10) A 1000 mL bag infused at the rate of $45 \mathrm{~mL} / \mathrm{h}$ using an infusion set with a drop factor of 10 .

$$
1000 \mathrm{~mL}\left(\frac{1 \mathrm{~h}}{45 \mathrm{~mL}}\right)=22.2 \mathrm{~h}
$$

11) A 1 L bag infused at the rate of $50 \mathrm{gtts} / \mathrm{min}$ using an infusion set with a drop factor of 15 .

$$
1 \mathrm{~L}\left(\frac{1000 \mathrm{~mL}}{\mathrm{~L}}\right)\left(\frac{15 \mathrm{gtts}}{\mathrm{~mL}}\right)\left(\frac{1 \mathrm{~min}}{50 \mathrm{gtts}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=5 \mathrm{~h}
$$

12) A 500 mL bag infused at the rate of $25 \mathrm{gtts} / \mathrm{min}$ using an infusion set with a drop factor of 20 .

$$
500 \mathrm{~mL}\left(\frac{20 \mathrm{gtts}}{\mathrm{~mL}}\right)\left(\frac{1 \mathrm{~min}}{25 \mathrm{gtts}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=6.7 \mathrm{~h}
$$

Answer the following:
13) A patient has an order for regular insulin at the rate of 18 units/hour. The solution is 100 mL with 100 units of regular insulin. An infusion set with a drop factor of 20 is being used. What will be the flow rate in $\mathrm{gtts} / \mathrm{min}$ ?

$$
\frac{18 \text { units }}{\mathrm{h}}\left(\frac{100 \mathrm{~mL}}{100 \mathrm{units}}\right)\left(\frac{20 \mathrm{gtts}}{\mathrm{~mL}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)=\frac{6 \mathrm{gtts}}{\mathrm{~min}}
$$

14) A patient has an order for a drug to be infused at the rate of $5 \mathrm{mcg} / \mathrm{kg} / \mathrm{min}$. A 500 mL bag contains 250 mg of the drug and the patient weighs 185 pounds. An infusion set with a drop factor of 20 is being used. What is the flow rate in gtts/min?
$185 \mathrm{lb}\left(\frac{1 \mathrm{~kg}}{2.2 \mathrm{lb}}\right)\left(\frac{5 \mathrm{mcg}}{\mathrm{kg} \mathrm{min}}\right)\left(\frac{500 \mathrm{~mL}}{250 \mathrm{mg}}\right)\left(\frac{1 \mathrm{mg}}{1000 \mathrm{mcg}}\right)\left(\frac{20 \mathrm{gtts}}{\mathrm{mL}}\right)=\frac{17 \mathrm{gtts}}{\mathrm{min}}$
15) A patient has an order for a drug to be infused at the rate of $25 \mathrm{mg} / \mathrm{kg} / \mathrm{h}$. A 1 L bag contains 10 g of the drug and the patient weighs 80 kg . An infusion set with a drop factor of 15 is being used. What is the flow rate in $\mathrm{gtts} / \mathrm{min}$ ?

$$
80 \mathrm{~kg}\left(\frac{25 \mathrm{mg}}{\mathrm{~kg} \mathrm{~h}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)\left(\frac{1 \mathrm{~L}}{10 \mathrm{~g}}\right)\left(\frac{1000 \mathrm{~mL}}{\mathrm{~L}}\right)\left(\frac{1 \mathrm{~g}}{1000 \mathrm{mg}}\right)\left(\frac{15 \mathrm{gtts}}{\mathrm{~mL}}\right)=\frac{50 \mathrm{gtts}}{\mathrm{~min}}
$$

In practice, you would probably change the 1 L bag to 1000 mL and the 10 g to $10,000 \mathrm{mg}$ before you started the problem. This would simplify things a bit and result with the following calculation.

$$
80 \mathrm{~kg}\left(\frac{25 \mathrm{mg}}{\mathrm{~kg} \mathrm{~h}}\right)\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)\left(\frac{1000 \mathrm{~mL}}{10,000 \mathrm{mg}}\right)\left(\frac{15 \mathrm{gtts}}{\mathrm{~mL}}\right)=\frac{50 \mathrm{gtts}}{\mathrm{~min}}
$$

